

Occam's Higgs: A Phenomenological Solution to the Electroweak Hierarchy Problem

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Abstract

We propose a phenomenological solution to the Electroweak hierarchy problem. It predicts no new particles beyond those in the Standard Model. The Higgs is arbitrarily massive and slow-roll inflation can be implemented naturally. Loop corrections will be negligible even for large cutoffs.

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The hierarchy problem [1] for the Electroweak component of the Standard Model (SM) has led to a number of theoretical conjectures towards its resolution, from Technicolor to Supersymmetry. All have encountered increasing difficulties with experiments. An alternative concept, deconstruction from a higher dimension, has been proposed [2, 3]. Based on this idea, a phenomenological solution to the hierarchy problem has been put forward in Ref.[4]. This solution invents *only* 30 new degrees of freedom (particles and interactions) to solve the problem vs., for example, 56 in Ref.[5] and from 126 up to thousands in supersymmetric approaches.

Why is it widely assumed that the mass of the Higgs necessarily implies the existence of (so) many additional degrees of freedom? Such a direction of thought is motivated by theoretical prejudices which, although they are highly attractive and have been beneficial in advancing understanding in the past, may nonetheless perhaps not be applicable here. As a result of this prejudice, even more modest, phenomenological approaches have not been fully explored. We delve into this viewpoint here.

In particular, we ask what is the minimal (hence, “Occam”) implication of the (apparently large) mass of the Higgs compared to expectations in the SM? We contend that the only things known with certitude are that the Higgs has a nonzero vacuum expectation value, $\langle vev \rangle$, and a possibly very large mass. We seek to represent this information phenomenologically.

We observe here that nothing at all new need be proposed, provided that one is willing to describe the Higgs sector as an effective theory¹ with unknown (and, we suspect, unknowable without much new data) underlying degrees of freedom. That is, we accept a non-renormalizable Higgs potential.²

We take the phenomenological constraints to be two-fold: (1) the $\langle vev \rangle$ of the Higgs should be stable, and (2) the mass of the Higgs should be arbitrarily large. These objectives can be straightforwardly achieved through nonlinearity and the abandonment of renormalizability as a constraint. We would be tempted to call what we present here an “effective field theory” but that terminology is already spoken for [8]. It has specific meanings in terms of relations to underlying physics and degrees of freedom.

¹For a general analysis in terms of renormalizable theories where loop corrections must be implemented, see Ref.[6]. Much additional work was stimulated by this paper.

²This attitude is similar in spirit to the “super-weak” line of thought developed in connection with CP-violation by Wolfenstein [7].

What we have in mind may be thought of as a possible (path integral) solution of an underlying field theory represented in terms of effective degrees of freedom. A source for the (composite) Higgs is introduced and path integration carried out to produce all of the n-Higgs vertex functions. These include all quantum corrections and fully describe all self-interactions of Higgs bosons. (We ignore vertex functions involving coupling to other observed degrees of freedom.) As such, the Lagrangians described below should have complex momentum dependences in the coefficients of the n-Higgs vertex functions. We approximate these crudely by simple constants.

An objection might be raised regarding the question of the constraint relating renormalization, mass, and unitarity. However, as has been observed before (see e.g., Ref.[9]), this relation is based on satisfying unitarity perturbatively. In fact, without self-shielding effects, unitarity is violated at a pole location unless the width of the particle involved is sufficiently large. Using an N/D method, Lee et. al. [10, 11] enforced unitarity without a perturbative requirement. They found that the effective Higgs' mass and width both increase significantly, but did not encounter any consistency problem. The effective interaction becomes strong and self-shielding to preserve the unitarity bound that MUST be satisfied. This phenomenon is familiar from the theory of Regge trajectory exchange [12] applied to strong interaction physics.

The $\langle vev \rangle$ may be fixed by replacing the SM Higgs by a nonlinear representation, analogous to that in the nonlinear sigma model. In the $O(n+1)$ sigma model, one replaces the invariant

$$\mathcal{L}_0 = (\sigma - \sigma_0)^2 + \sum_{i=1}^n \pi_i^2, \quad (1)$$

where σ_0 is the sigma field $\langle vev \rangle$, by its nonlinear equivalent [13]

$$\mathcal{L}_1 = \frac{\sum_{i=1}^n \pi_i^2}{\sqrt{(\sigma_0)^2 + \sum_{i=1}^n \pi_i^2}}, \quad (2)$$

thereby disallowing any fluctuations of the σ field by fiat. The cost is the appearance of a nonrenormalizable theory, which is then viewed as an effective field theory approximation to some underlying one. Formally, this is described by a group ratio, $SO(n+1)/SO(n)$ in this example.

A slightly less drastic approach allows for fluctuations of the σ field, again at the cost of introducing nonrenormalizable interactions. In particular, let the Lagrangian density

$$\mathcal{L}_2 = \left\{ \sigma^2 + \sum_{i=1}^n \pi_i^2 - (\sigma_0)^2 \right\}^2 \quad (3)$$

be replaced by

$$\mathcal{L}_3(x) = V(x) = \Lambda^4 \left\{ 1 + N \left[e^{a(x-c)} - e^{b(x-c)} \right] \right\}, \quad a > b, \quad (4)$$

$$N = \left(\frac{b}{a-b} \right) \left(\frac{a}{b} \right)^{a/(a-b)}, \quad (5)$$

$$x \equiv \sqrt{(\sigma^2 + \sum_{i=1}^n \pi_i^2)}, \quad (6)$$

where Λ sets the arbitrary mass scale. (Note, therefore, that $(\Lambda, x, c, 1/a, 1/b)$ all have units of mass.) In Figs.1 and 2 we show $V(x)/\Lambda^4$ vs. x for $(a = 10, b = 9, c = 5)$ and $(a = 100, b = 99, c = 5)$, respectively (where the values of a and b are defined in units of Λ^{-1} and those of c are in terms of Λ).

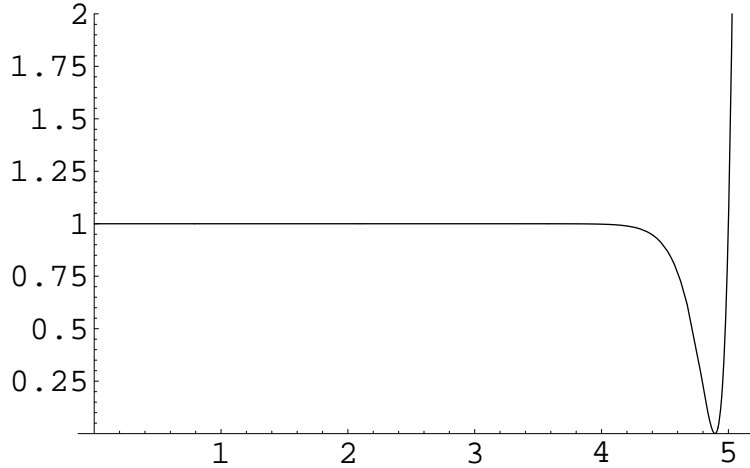


Figure 1: $V(x)/\Lambda^4$ vs. x for $(a = 10, b = 9, c = 5)$.

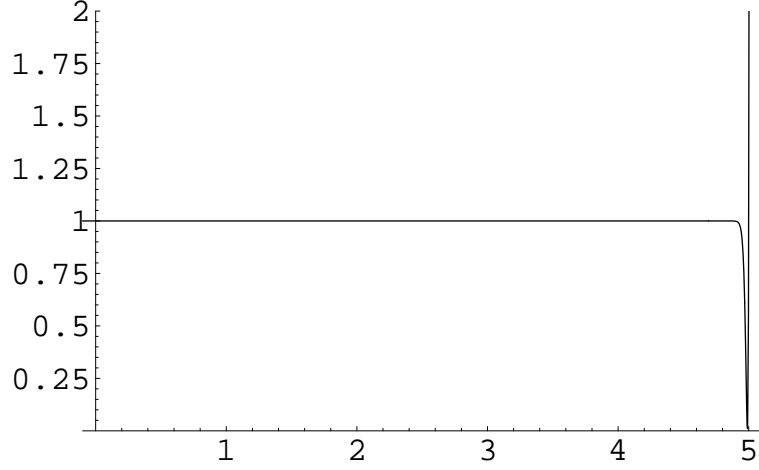


Figure 2: $V(x)/\Lambda^4$ vs. x for $(a = 100, b = 99, c = 5)$.

With only four parameters, (Λ, a, b, c) , $V(x)$ can fix the value of $\langle vev \rangle$ completely, independently of the curvature about the minimum in the field potential. As the latter determines the mass of the field representing fluctuations about the $\langle vev \rangle$, that mass may now be chosen freely to match the results of experiments.

Specifically, the $\langle vev \rangle$ is given by the value of x_0 where

$$0 = \left(\frac{\partial V}{\partial \sigma} \right)_{x=x_0} = V'(x_0) \left(\frac{\partial x}{\partial \sigma} \right)_{x=x_0}. \quad (7)$$

Since at x_0 , with $\pi_i = 0$,

$$\left(\frac{\partial x}{\partial \sigma} \right)_{x=x_0} = 1, \quad (8)$$

one has

$$\langle vev \rangle = x_0 = c - \frac{\ln(a/b)}{(a-b)}. \quad (9)$$

Note also that

$$\lim_{(a-b) \rightarrow \delta \ll (a,b)} x_0 = \left(c - \frac{1}{b} \right) \xrightarrow{b \rightarrow \infty} c. \quad (10)$$

The mass-squared is then given by

$$\begin{aligned}
m^2 &= \left(\frac{\partial^2 V}{\partial^2 \sigma} \right)_{x=x_0} = V''(x_0) \\
&= \Lambda^4 N \left[a^2 e^{a(x_0-c)} - b^2 e^{b(x_0-c)} \right] \\
&= ab \Lambda^4,
\end{aligned} \tag{11}$$

since $V'(x_0) = 0$.

For any of Λ , a , and b separately arbitrarily large, the value of the mass can be pushed to arbitrarily large values, overwhelming quadratically divergent corrections from additional interactions whatever the scale of the cutoff. From our point of view this is irrelevant anyway, as the theory is the approximate solution to the full result of path integration over the (unknown) underlying degrees of freedom.

It is straightforward to carry out this procedure for the weak isoscalar Higgs isodoublet field of the SM. The result continues to suggest that underlying degrees of freedom exist, but abandons the notion that those of the SM necessarily represent fundamental quantities. Implicitly this applies to the quarks, leptons, and gauge bosons as well, as they may or may not themselves be fundamental degrees of freedom. Since string theory and even supersymmetry allow for this possibility, the suggestion made here should not be considered radical.

Interestingly, potentials similar in form to that suggested here appear in some approaches to vacuum stabilization in string theory. [14, 15] Further, potentials based on exponential forms have often been suggested within the context of the cosmological constant problem and quintessence [16].

A side benefit of this kind of potential is that it is of the type that is required by studies of inflation [17] as a solution to the flatness problem of the Universe. This occurs because

$$V'(0) = \Lambda^4 N \left[ae^{-ac} - be^{-bc} \right] \tag{12}$$

behaves, in the limit $b \rightarrow a$, as

$$V'(0) \rightarrow -a(ac-1) e^{-(ac-1)} \Lambda^4, \tag{13}$$

which is arbitrarily exponentially small as $c \rightarrow \infty$. Note that this does not impinge upon the mass-squared value in Eq. (11).

In conclusion, we ask what if the electroweak loop corrections were not already included in the conjectured approximate solution that we present for the path integral of whatever the underlying theory may be? We answer that, with this potential and for sufficiently large m^2 , electroweak loop corrections could be maintained as negligible even for cutoffs much larger than the 1 – 10 TeV generally considered at present.

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